

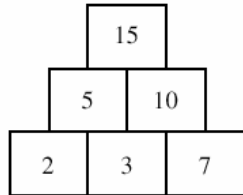
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## Number Towers

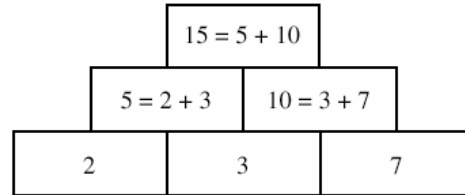
This problem gives you the chance to:

- form and solve equations obtained from a number pattern
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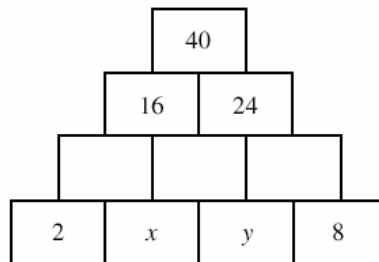
An addition number tower is shown below.



In this tower, each number is the sum of the two numbers just below it.

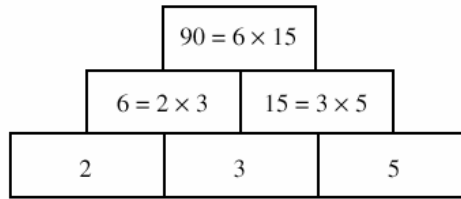
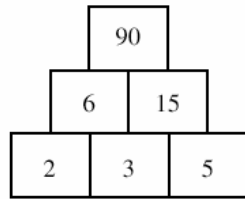


Marcie makes a bigger number tower:

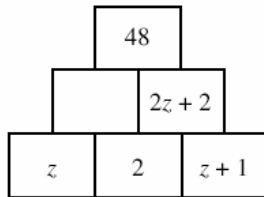


1. Complete the number tower by writing algebraic expressions in the empty boxes.
2. Use the completed number tower to show that  $2x + y = 14$  and  $x + 2y = 16$ .
3. Find values of  $x$  and  $y$  that satisfy both equations:  $2x + y = 14$  and  $x + 2y = 16$ .

Marcie also designs a **multiplication** number tower.



Look at the multiplication tower below.



4. Complete the multiplication tower by writing an algebraic expression in the empty box.

5. Use the completed multiplication tower to show that  $4z^2 + 4z = 48$ .

6. Find the possible values of  $z$  in  $4z^2 + 4z = 48$ .

Number Towers		Test 9 Form A Rubric	
The core elements of performance required by this task are: • form and solve equations obtained from a number pattern Based on these, credit for specific aspects of performance should be assigned as follows:		Points	Section Points
1. Gives correct answers as: $2 + x, x + y, y + 8$ All three correct answers: 2 points <i>Partial credit:</i> Two correct answers: 1 point		2 (1)	2
2. Gives correct answers as: $2 + 2x + y = 16$ therefore $2x + y = 14$ $x + 2y + 8 = 24$ therefore $x + 2y = 16$		1 1	2
3. Gives correct answers as: $x = 4$ $y = 6$		1 1	2
4. Gives correct answer as: $2z$		1	1
5. Gives correct answer as: $2z(2z + 2) = 48$ therefore $4z^2 + 4z = 48$		1	1
6. Gives correct answers as: $z^2 + z - 12 = 0$ $(z + 4)(z - 3) = 0$ $z = 3$ or $-4$ <i>Partial credit for only one solution.</i>		2 (1)	2
<b>Total Points</b>			<b>10</b>

## Looking at Student Work – Number Towers

Students did not do well on Number Towers. Of the 150 papers reviewed, 56 did not attempt the problem and an additional 51 scored no points on the task.

Student A shows evidence of understanding the patterns within the number towers and being able to represent each space symbolically. This is shown clearly in the logic structure used for the proofs in part 2. This student understands the tools of algebra and can solve simultaneous equations with two unknowns in part 3. The student also uses the quadratic equation to solve for the missing roots in part 6.

**Student A**

NO RESPONDE

2

1. Completé la torre de números escribiendo expresiones algebraicas en los recuadros vacíos.

$x+z$        $x+y$        $y+z$

2. Utilice la torre de números completada para mostrar que  $2x + y = 14$  y  $x + 2y = 16$ .

$2x + y = 14$

$x + z + x + y = 16$

$2x + y = 16 - z$

$2x + y = 14$

$x + y + y + z = 24$        $x + z + y = 16$

$x + 2y + z = 24$

$x + 2y = 24 - z$

$x + 2y = 16$

3. Encuentre los valores de  $x$  y  $y$  que satisfacen ambas ecuaciones:  $2x + y = 14$  y  $x + 2y = 16$ .

$2x + y = 14$

$x + 2y = 16$

$2x + y = 14$

$x + 2y = 16$

$y = 14 - 2x$

$x + 2(14 - 2x) = 16$

$x + 28 - 4x = 16$

$-3x + 28 = 16$

$-3x = 16 - 28$

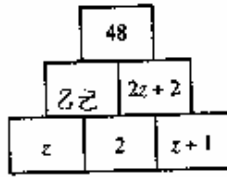
$-3x = -12$

$x = \frac{-12}{-3}$

$x = 4$

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Página 8      Torres de números      Prueba B, Página A       $y = 6$



4. Complete la torre de multiplicación escribiendo una expresión algebraica en el recuadro vacío.

5. Utilice la torre de multiplicación to show that  $4z^2 + 4z = 48$ .

$$\begin{aligned} z(z+4) &= 48 \\ 4z^2 + 4z &= 48 \end{aligned}$$



6. Encuentre los valores posibles de  $z$  en  $4z^2 + 4z = 48$ .

$$4z^2 + 4z - 48 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-48)}}{2(4)} \\ x &= \frac{-4 \pm \sqrt{16 + 768}}{8} \\ x &= \frac{-4 \pm \sqrt{784}}{8} \\ x &= \frac{-4 \pm 28}{8} \end{aligned}$$

$x = 3$   
 $x = -4$

$x = \frac{-4 + 28}{8} = 3$  } 10

Student B demonstrates the same understandings, but chooses to use factoring to solve for the roots in part 6 of the task.

### Student B

5. Use the completed multiplication tower to show that  $4z^2 + 4z = 48$ .

$$2z(2z+2) = 48$$

$$2z^2 + 4z = 48$$

6. Find the possible values of  $z$  in  $4z^2 + 4z = 48$ .

$$4z^2 + 4z - 48 = 0$$

$$z^2 + z - 12 = 0$$

$$(z-3)(z+4) = 0$$

$$z = 3 \checkmark \text{ or } -4 \checkmark$$

The most challenging demand of the task was to use algebra to prove why certain algebraic expressions were true. Students did not understand how to do that piece particularly for part 2. However many students with weaker skills good solve part 3 and 5 using guess and check or substitution. See the work of Student C.

### Student C

3. Find values of  $x$  and  $y$  that satisfy both equations:  $2x + y = 14$  and  $x + 2y = 16$ .

$$x = 4 \quad y = 6$$

$$\begin{aligned} 2 \cdot 4 + 6 &= 14 \\ 8 + 6 &= 14 \\ 14 &= 14 \end{aligned}$$

$$\begin{aligned} 4 + 2 \cdot 6 &= 16 \\ 4 + 12 &= 16 \\ 16 &= 16 \end{aligned}$$

2

5. Use the completed multiplication tower to show that  $4z^2 + 4z = 48$ .

$$2z(2z+z)$$

$$4z^2 + 4z = 48$$

$$4(3)^2 + 4(3) = 48$$

$$4(9) + 12 = 48$$

$$36 + 12 = 48$$

6. Find the possible values of  $z$  in  $4z^2 + 4z = 48$ .

$$48 = 48$$

$$4(3)^2 + 4(3) = 48$$

$$4(9) + 12 = 48$$

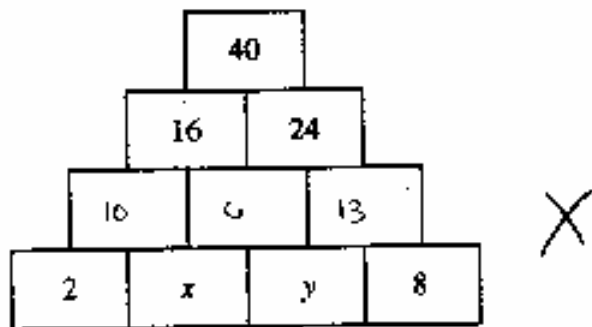
$$36 + 12 = 48$$

$$48 = 48$$

$$z = 3$$

More than half the students who responded to Number Towers put numbers into the blank boxes in the number towers instead of algebraic expressions. Student D picks numbers, which will add to 16, but then puts in an incorrect number for the third box. In doing guess and check in part 3, the student does not understand the concept that the two equations represent conditions that must both be true. The student just generates different values for  $x$  and  $y$  to satisfy each equation. This is a very fundamental issue in working with algebra.

### Student D



Complete the number tower by writing algebraic expressions in the

$$10 + 6 = \boxed{16} + 13 = \boxed{29}$$

Complete the completed number tower to show that  $2x + y = 14$  and  $x + 2y = 14$

$$2(5) + 4 = 14$$

$$6 + 2(5) = 14$$

Find values of  $x$  and  $y$  that satisfy both equations:  $2x + y = 14$  and  $x + 2y = 14$

$$2x + y = 14$$

$$x = 5$$

$$y = 4$$

$$x + 2y = 14$$

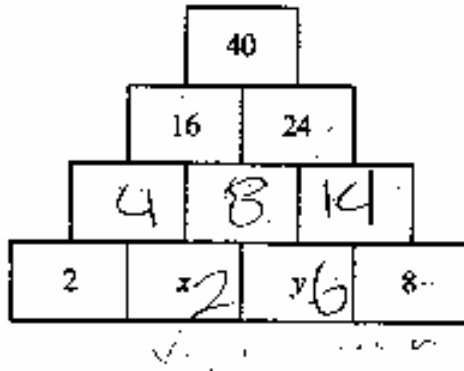
$$x = 6$$

$$y = 5$$



Student E generate possible values for  $x$  and  $y$  in the bottom row of the tower, then uses the addition rule to fill in the missing numbers in the second row. The student does not appear to notice that the numbers in the second row will not produce the sums given for the third. Student E does not respond to any further parts of the task.

### Student E



### Teacher Notes:

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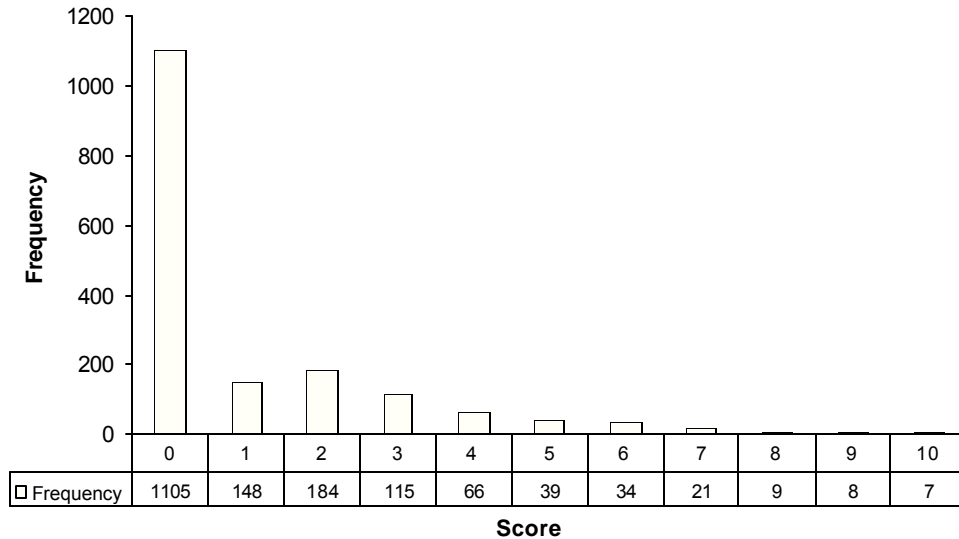
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Grade 9 – Number Towers

### Number Towers

Mean: 1.09, S.D.: 1.87



Score:	0	1	2	3	4	5	6	7	8	9	10
% ≤	63.7%	72.2%	82.8%	89.4%	93.2%	95.4%	97.4%	98.6%	99.1%	99.6%	100.0%
% ≥	100.0%	36.3%	27.8%	17.2%	10.6%	6.8%	4.6%	17.2%	1.4%	0.9%	0.4%

The maximum score available for this task is 10 points.

The cut score for a level 3 response is 5 points.

Some students (about 36%) could find one correct solution to satisfy the equations in part 3 or could fill in the correct symbolic expression for the multiplication tower. Less than 10% of the students met standard on this task. Some of these students managed to do this use guess and check strategies to solve for unknowns. More than 60% of the students scored zero on this task. About half of the students who scored zero attempted the task.

## Number Towers

<b>Points</b>	<b>Understandings</b>	<b>Misunderstandings</b>
<b>0</b>	More than 60% of the students scored no points on this task. About half of them attempted the task.	Students who attempted the task did not understand how to use symbolic notation to continue the pattern in the towers. They used numbers instead of expressions to fill in the blank boxes. While some of the numbers may have fit the pattern, there were still flaws in the solution.
<b>2</b>	Students with this score could usually use a guess and test or substitution strategy to find the correct values for the equations in part 3. A few students could write symbolic expressions in the number towers.	
<b>3</b>	Students could fill in the symbolic expressions for both number towers.	
<b>5</b>	Students could use substitution to find the unknowns in 3 and the positive root in 6. They could fill in the multiplication tower and use the distributive property to make the proof for part 6.	Only 11 of the 150 students used algebra to solve part 3. Another 2 students attempted to use algebra in part 3.
<b>7</b>	Students with this score could not find the negative root for part 6 and made one other error in some other part of the task.	2 students used the quadratic equation successfully in part 6, another 2 attempted to use the quadratic formula. 2 students used factoring successfully in part 6 and 7 attempted to use factoring.
<b>10</b>	Students could use symbolic notation and convert among equivalent expressions to form a proof. Students could solve two equations for two unknowns using guess and check or algebra. Students could find roots for a quadratic equation using factoring or the quadratic formula.	

Based on teacher observations, this is what Algebra One students seemed to know and be able to do:

- Use guess and check or substitution to solve for two unknowns in two equations.
- Write symbolic multiplication expressions.
- Simplify a multiplication expression using the distributive property.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Writing symbolic addition expressions
- Checking their solutions to see if they fit all the conditions or parameters of the task
- Using symbolic expressions to form equivalent expressions
- Solving two equations for two unknowns
- Using factoring or the quadratic formula to find missing roots

### Questions for Reflection on Number Towers

- Did your students use numbers or symbolic expressions to fill in the number towers? If they used numbers, did the numbers fit all parameters of the tower, e.g. did the numbers lead to the correct final solution? What are your classroom norms around checking having students check their work? What do you think they didn't understand about the pattern?
- What opportunities have your students had to use algebra to do a proof? What types of experiences or problems might help them develop these skills? What would you have liked to see in a complete and thorough explanation for part 2? What skills would be needed for that solution?
- While guess and check or substitution is a useful tool for solving problems, at this grade level students need to have their skills expanded to use more efficient strategies. What types of questions can help students see the application of the algebra for solving problems? How do you provide learning experiences to help students transition from guess and check to using algebra?

Looking at student work, how many of your students in part 3:

Did not attempt this part?	Used guess and check or substitution?	Used algebra to solve?

In part 6, how many of your students:

Did not attempt this part?	Used the quadratic formula?	Used factoring?

- Do you think students would have done better on this problem if they had more time? Where might this problem fit into your curriculum to test for these understandings?

**Teacher Notes:**

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**Instructional Implications:**

Students at this grade level need frequent opportunities to describe situations in symbolic notation. It is important that they not just use expressions provided for them for symbol manipulation. Students also need to develop their logic and have opportunities to use algebra to prove conjectures or rectify different solutions or formulae. Students should know that given two equations, the common variables stand for the same values. Students should be able to find positive and negative solutions for quadratic equations and have a variety of tools to help them.

**Teacher Notes:**

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<b>Performance Assessment Task</b>				
<b>Number Towers</b> <b>Grade 9</b>				
The task challenges a student to demonstrate understanding of the concepts of algebraic properties and representations. A student must make sense of the meaning of equivalent forms of expressions, equations, inequalities or relations in different contexts; this one is a number pattern. A student must be able to write equivalent forms of equations, inequalities and systems of equations and solve them when given pieces of information. A student must be able to use symbolic algebra to represent and explain mathematical relationships derived from the number patterns. Given specific constraints, students must be able to use algebra to determine the solution to the specified questions.				
<b>Common Core State Standards Math - Content Standards</b>				
<b>High School – Algebra – Creating Equations</b>				
<b>Create equations that describe numbers or relationships.</b>				
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.				
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>				
<b>Common Core State Standards Math – Standards of Mathematical Practice</b>				
<b>MP.6 Attend to precision.</b>				
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.				
<b>MP.8 Look for and express regularity in repeated reasoning.</b>				
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$ , $(x-1)(x^2+x+1)$ , and $(x-1)(x^3 +x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.				
<b>Assessment Results</b>				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
<b>Grade Level</b>	<b>Year</b>	<b>Total Points</b>	<b>Core Points</b>	<b>% At Standard</b>
9	2003	10	5	7 %